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Multiple spatially-overlapping sets can be enumerated in parallel

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Abstract

A system for non-verbally representing the approximate number of items in visual and auditory arrays has been documented in multiple species, including humans. While many aspects of this approximate number system are well-characterized, the fundamental questions remain unanswered of how attention selects which items in a scene to enumerate, and how many enumerations can be computed simultaneously. Here we show that when presented an array containing different numbers of spatially-overlapping dots of many colors, human adults can select and enumerate items on the basis of shared color and can enumerate approximately 3 color subsets from a single glance. This 3-*set* limit converges with previously-observed 3-*item* limits of parallel attention and visual short-term memory. This suggests that participants can select a subset of items from a complex array as a single individual set, which then serves as the input to the approximate number system.

Many species, including humans, can represent the approximate number of items in visual or auditory arrays without verbally counting (e.g., Dehaene, Dehaene-Lambertz & Cohen, 1998; Feigenson, Dehaene, & Spelke, 2004 for reviews). Various aspects of this enumeration ability have been characterized, including its developmental progression (Lipton & Spelke, 2003; Siegler & Opfer, 2003) and its likely location in a region of the intraparietal sulcus in humans (Piazza, Izard, Pinel, LeBihan, & Dehaene, 2004; but see Shuman & Kanwisher, 2004).

This "approximate number system" allows for the recognition of numerical quantities. But which items does it actually enumerate? A glance at any natural scene reveals the potentially limitless sets of items that might be enumerated (e.g., the trees, buildings, pedestrians, etc.). Yet at any given time we are only aware of the numerosity of a few of these possible subsets; indeed, often we are unaware of numerosity altogether. This raises several questions regarding the nature of numerical processing.

First, what is the process by which some items in a scene are selected for enumeration while others are ignored? Previous research on the approximate number system has required participants to enumerate only a single set of items (e.g., all of the dots on a screen). The role of attentive selection in forming number representations is therefore not understood.

Second, if enumeration of a subset is possible, can multiple subsets be enumerated at once? For example, could one look outside and simultaneously represent both the number of trees and the number of pedestrians without verbally counting? While this question has never been explicitly tested, previous work has found that rats treat stimuli with different perceptual properties (light flashes and auditory tones) as a single stimulus set, such that two flashes and two tones were treated as 4 events (Church & Meck, 1984).

To address both of these questions, we created a task in which participants saw briefly flashed arrays containing from 1 to 35 dots. Participants had to enumerate either all of the items or just a subset defined by a common color. Items were completely spatially intermixed. On some trials participants were informed in advance which subset they were to enumerate (Probe Before trials). On other trials participants were not told which subset to enumerate until after the array had been flashed (Probe After trials).

In Experiment 1 we used Probe Before trials to ask whether participants could attentively select which items to enumerate. We also compared performance on Probe Before and Probe After trials to determine the number of subsets participants could enumerate in parallel from a single array. In Experiments 2 and 3 we confirmed that participants were responding to discrete number and not to continuous dimensions such as total area or circumference.

INSERT FIGURE 1 ABOUT HERE

Experiment 1

Participants

Ten naive adults with normal vision received course credit for participation.

Materials and Apparatus

Participants viewed 450 trials on a Macintosh iMac computer (CRT monitors with viewable area measuring 29.5 X 22.5 cm). Viewing distance was unconstrained, but averaged approximately 50 cm. The diameter of each dot subtended approximately 1 degree of visual angle from a viewing distance of 50 cm.

Design and Procedure

On each trial, participants saw a 500ms display containing between 1-35 dots of between 1-6 colors (red, blue, yellow, green, cyan, magenta). The number of dots in each subset was randomly determined with the constraint that the target subset was smaller than at least one distractor subset on half of the trials. This made strategies such as "attend only the largest set" ineffective. Dot position was randomly determined with the constraint that dots never overlapped. The 450 trials were presented in a single block in randomized order. A single set or the superset was probed on each trial. Participants typed their numerosity judgments on the numeric keypad.

Results and Discussion

Previous research shows that when the approximate number system is engaged, both the mean and standard deviation of numerical estimates increase linearly as a function of the number of items to be enumerated, resulting in a constant coefficient of variance ranging from approximately .14 to .35 in human adults (Cordes, Gelman, & Gallistel, 2001;Whalen, Gallistel, & Gelman, 1999). We first asked whether participants' responses accorded with this pattern on Probe Before trials containing dots of just a single color (1-Color Probe Before trials), in which no attentive selection was required. A 1-Color Probe Before trial might contain a probe telling participants to enumerate the blue dots, followed by an array containing 15 blue dots and no other dots. Means were constructed by binning the trials from every participant for target numerosities 7-30 because only these target numerosities could appear for every trial type (i.e. 1-6 Colors). Figure 2A shows that both the mean and the SD of participants' responses increased linearly: mean, $t\,(6) = 24.064$, $p < .001$, $p_{rep} = .999$, $d = 19.65$; SD, $t\,(6) = 5.602$, $p < .001$, $p_{rep} = .988$, $d = 4.57$. The average coefficient of variance was .17. This demonstrates that our task engaged the approximate number system.

INSERT FIGURE 2 ABOUT HERE

Next we asked whether selective attention is required prior to enumeration. If participants can attend to and enumerate just a subset of the array, then the identical performance signature should be observed on Multiple-Color Probe Before trials as on 1-Color Probe Before trials. A Multiple-Color Probe Before trial might contain a probe telling participants to enumerate the blue dots, followed by an array containing red, blue, and green dots. Figure 2B shows that on these trials the signature of the approximate number system was again observed. Both the mean and the SD of responses increased linearly: mean, t (6) = 17.208, $p <$ .001, p_{rep} = .999, d = 14.05; SD, t (6) = 15.702, $p <$ .001, p_{rep} = .999, d = 12.82, and the average coefficient of variance was .21. Indeed, the accuracy of participants' estimates did not change as the number of distractor sets in the display increased, resulting in a nonsignificant linear regression, showing that attention was efficient at selecting which subset to enumerate (*for example see* Figure 2C black bars: t (4) = 1.031, p = .361).

Finally, we asked whether adults can enumerate multiple subsets of dots in parallel. Probe Before and Probe After trials formed yoked pairs: for each Probe Before trial there was a Probe After trial containing the identical number of colors and the identical number of dots in each color subset. If participants could enumerate *all* of the color subsets in the display, then error on Probe After trials should not differ from error on Probe Before trials, despite the fact that on Probe After trials participants did not know which color subset to enumerate until after the display had disappeared. Representing only some of the color subsets would result in the participant sometimes *not* knowing the correct answer to the probe. This would lead to a difference in performance between Probe Before and Probe After trials. For each participant, planned *t*-tests compared error on Probe Before to error on Probe After trials (e.g. Figure 2C). The number of colors at which error on Probe After trials became significantly higher than that on Probe Before trials indicated the number of sets the participant could encode and recall from a single display.

We found that most participants encoded 3 sets from a single display (mean = 2.6, range = 2-4). Figure 2C displays the error pattern for the participants in Experiment 1 who enumerated 3 sets in parallel. The results show that participants always selected the superset of all dots for enumeration on Probe After trials, irrespective of the number of colors in the display. Thirteen percent of trials prompted participants to report the total number of dots in the display (All Dots). Error did not differ on All Dot trials for Probe Before versus Probe After for any participant, regardless of the number of colors shown (all $p > .05$). The superset of All Dots appears to have been represented as a single set, rather than as the sum of subsets, for two reasons. First, participants succeeded at enumerating All Dots on Probe After trials containing up to 6 color subsets. This was beyond the number of separate color subsets that any participant could enumerate. Second, on All Dot Probe After trials participants had a constant reaction time regardless of how many colors comprised the array, suggesting that additional colors added no time to the computation of numerosity (6 Trial Type Repeated Measures ANOVA, $F(5,45) = 1.150$, $p = .348$). Thus, for participants who encoded three sets in parallel, these sets were the superset of All Dots and two color subsets (Figure 2C).

That participants were limited to enumerating approximately 3 sets converges with the 3-item limit that has been observed in studies of object-based attention (Pylyshyn & Storm, 1988; Scholl, 2001) and visual short-term memory (Alvarez & Cavanaugh, 2004; Luck & Vogel, 1998). This agreement between the set-based limit we observed here and previously observed item-based limits suggests that each set of dots in Experiment 1was attended and stored as an *individual*, despite the fact that the individual dots within each set remained available for enumeration by the approximate number system. Previous work has shown that adults can attend at most 3 or 4 individuals, where individuals are roughly bounded, coherent objects (Scholl, 2001) or spatially-bounded groups (e.g., flocks of birds; vanMarle & Scholl, 2003; Wynn, Bloom, & Chiang, 2002). The present work shows that even a non-object entity that does not occupy a bounded spatial location, such as “the set of red dots” spatially intermixed with other dots, can function as an individual for attention and short-term memory, and that “approximate number” can be stored as a feature of this individual.

Experiments 2 & 3

To ensure that participants’ responses were based on number rather than on any continuous dimension correlated with number, we replicated Experiment 1 controlling for area (Experiment 2) and contour length (Experiment 3), two dimensions to which infants have shown sensitivity given similar visual arrays (Clearfield & Mix, 1999; Feigenson, Carey, & Spelke, 2002).

Method

Twenty new naïve adults with normal vision received course credit for participation (n = 10 for each experiment). Methods were identical to Experiment 1 except that each color subset was constrained to have either a total cumulative area of 26,214 pixels (Experiment 2) or a total cumulative perimeter (i.e. cumulative circumference) of 667 pixels for every trial (Experiment 3) (Figure 1B). Thus, total area or circumference no longer co-varied with number. All items were circular, but of varying sizes. The size of each dot within a color subset was randomly selected from a range with a lower-upper bound of ±50% of the average size for a set (e.g. 26,214 pixels divided by the number of dots in the set). Thus, while total area or circumference was constant for every set, individual dot sizes varied to discourage the use of dot size as a cue to numerosity.

Results and Discussion

The pattern of results was the same as that in Experiment 1. On 1-Color Probe Before trials, both the mean and SD of responses increased linearly: Experiment 2 mean, t (6) = 13.673, $p < .001$, p_{rep} = .999, d = 11.16; SD, t (6) =2.504, $p < .05$, p_{rep} = .921, d = 2.04; coefficient of variance = .34; Experiment 3 mean, t (6) = 12.708, $p < .001$, p_{rep} = .999, d = 10.38; SD, t (6) = 9.224, $p < .001$, p_{rep} = .997, d = 7.53, coefficient of variance was .34. A similar pattern was observed on Multiple-Color Probe Before trials: Experiment 2 mean, t (6) = 23.216, $p < .001$, p_{rep} = .999, d = 18.96; SD, t (6) = 10.633, $p < .001$, p_{rep} = .998, d = 8.68, coefficient of variance = .31; Experiment 3 mean, t (6) = 6.125, $p < .001$, p_{rep} = .991, d = 5.00; SD, t (6) = 7.686, $p < .001$, p_{rep} = .995, d = 6.28, coefficient of variance = .39. This indicates that participants were able to select a subset of items that shared a common color and enumerate these items even when total surface area or circumference did not co-vary with number. The coefficient of variance observed in Experiments 2 and 3, while within the acceptable range for human adults, was higher than that observed in Experiment 1. This difference may indicate that participants in Experiment 1 were relying on multiple cues (e.g. number and area), while in Experiments 2 and 3 participants had to rely only on number. New experiments in our lab are testing this hypothesis.

As in Experiment 1, we compared each participant's performance on Probe Before versus Probe After trials in order to determine the number of sets that each participant could enumerate in parallel. We again found that most participants enumerated approximately 3 sets from a single display: Experiment 2 mean = 3.4, range 2-6; Experiment 3 mean = 3.2, range 2-5. Given the similar pattern of performance across our 3 experiments, the data can be collapsed to show the average number of subsets participants were able to enumerate. Figure 2D shows that this number was 3.

General Discussion

Our results specify at least two critical new aspects of numerical processing. First, and most surprisingly, our results demonstrate the ability to enumerate multiple sets of items in parallel. This 3-set limit converges with the 3-item limits of visual attention (Pylyshyn & Storm, 1988; Scholl, 2001) and visual short-term memory (Alvarez & Cavanaugh, 2004; Luck & Vogel, 1988). Our results suggest that about 3 individual sets can be stored in short-term memory, and that "approximate number" can be stored as a feature of each set.

Second, our results suggest that attentive selection is necessary prior to enumeration. Participants had no difficulty enumerating spatially intermixed subsets within a scene, so long as these subsets were selectable on the basis of a common early visual feature such as color. It remains to be tested whether other early visual features like size or orientation can also serve as the basis for the selection of to-be-enumerated items.

Considered from a broad perspective, our results illustrate the degree to which items in a visual scene can be hierarchically represented. For example, a single display of 12 blue, 5 red, 1 green, 3 magenta, and 7 yellow dots can be represented as 1 array, 5 groups, or 28 separate dots. At one level "the set of blue dots" may be selected and stored as a single individual. At another level "the set of blue dots" may be treated as 12 distinct items, available for enumeration by the approximate number system. This highlights a hierarchical coding of "set" and "individual" that is important for all mathematical concepts. Furthermore, our results indicate that the notion of a set may operate prior to enumeration by the approximate number system in adults, converging with evidence that "set" may be an important concept early in child development (Feigenson & Halberda, 2004).

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Figure Legends

Figure 1: (A) A 2-Color Probe After trial from Experiment 1. (B) Sample stimulus arrays from Experiments 1-3 drawn to scale. Example trials can be viewed at *http://www.psy.jhu.edu/~halberda/demos.html*

Figure 2: (A) Performance on 1-Color Probe Before trials from Experiment 1 exhibits the signature of the approximate number system: the mean and SD of responses increase linearly as a function of the number probed. (B) Performance on the Multiple-Color Probe Before trials from Experiment 1 also shows the signature of the approximate number system. (C) Error in numerical estimates for participants in Experiment 1 who succeeded at enumerating three sets in parallel. Error on Probe Before trials was compared to error on Probe After trials (±SE), * indicates $p < .05$. Error on Probe Before trials did not increase as the number of color subsets increased (black bars). In contrast, error on Probe After trials abruptly increased when 3 or more color subsets were displayed (white bars). For participants who enumerated four sets in parallel, the pattern was the same except that the first significant difference in performance occurred on trials containing four colors in the display, not three. Thus, a categorical increase in error on Probe After trials indicated the number of sets each participant enumerated in parallel. (D) Histogram of the number of sets each participant enumerated in parallel in Experiments 1-3 (mean = 3.1).

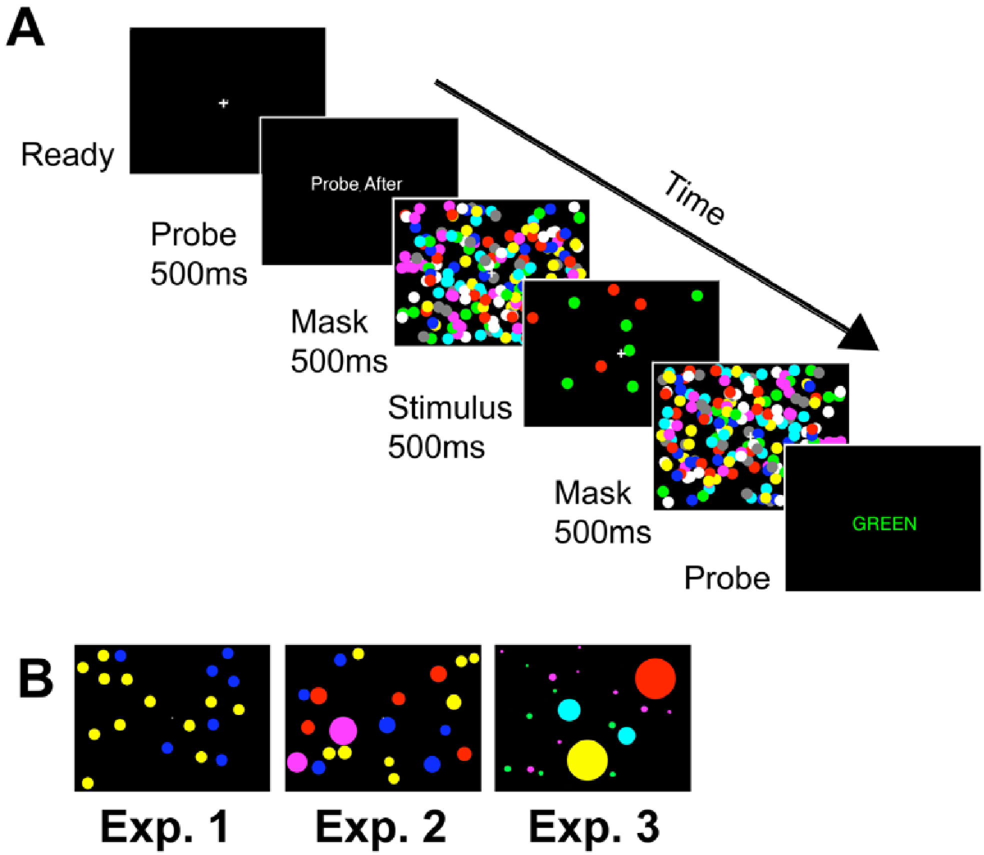
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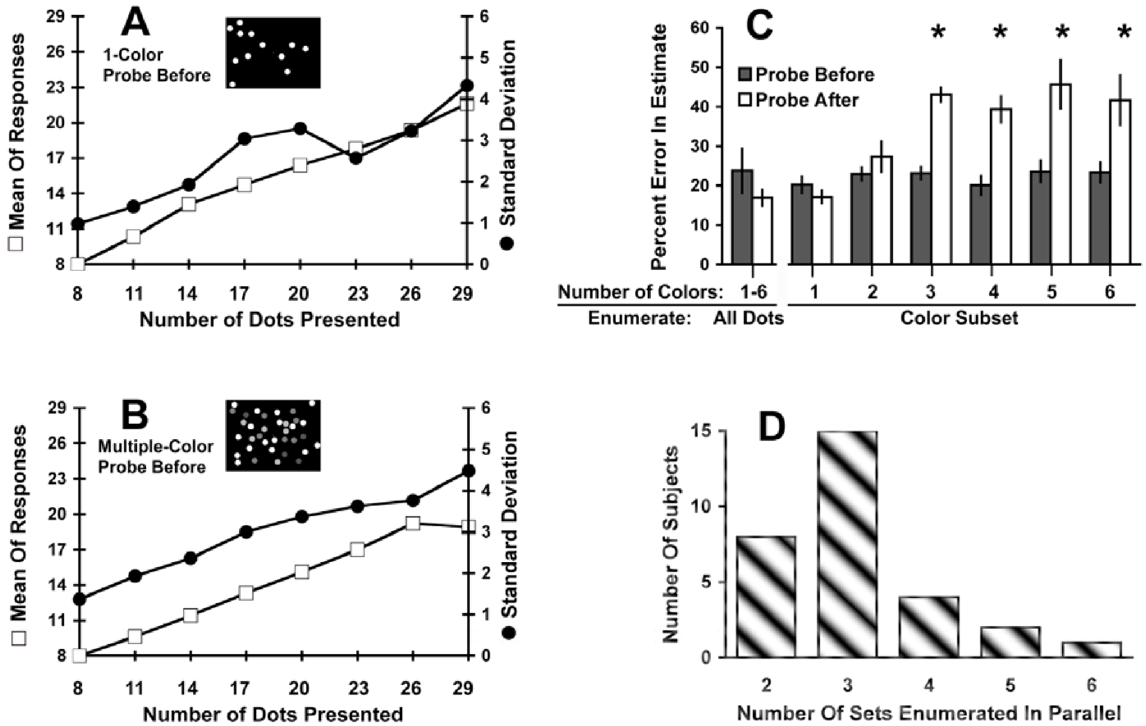
Figure 1


A
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Standard Deviation
Number of Dots Presented
B
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Probe Before
Mean Of Responses
Standard Deviation
Number of Dots Presented
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Percent Error In Estimate
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Figure 2